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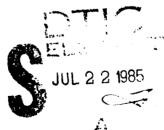
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A MODIFIED REPAIR STRATEGY FOR TWO-COMPONENT

SYSTEMS WITH REVEALED AND UNREVEALED FAULTS

Norman L. Johnson and Samuel Kotz

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FOR TWO-COMPONENT SYSTEMS WITH REVEALED AND UNREVEALED FAULTS

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ABSTRACT

Some consequences of a modified repair system for Phillips' (J.
Appl. Prob. 18, 652-659, Rel. Engineering, 2, 221-231) model for a two-
component system are discussed. In the original model, both components
are repaired whenever a revealed fault occurs; in the modified model
only faulty components are repaired. Specifically (i) the distribution
of time from the initial state up to discovery of an unrevealed fault,
(ii) the expected proportion of time during which there exists an
unrepaired fault, and (iii) the distribution of number of revealed
faults up to and including the one which leads to a discovery of an
unrevealed fault, are obtained. The theory is illustrated by examples,
based on specific distributions for the times between repairs and occurrences
of the two types of faults. A characterization of the exponential
distribution is indicated.

Key Words. Exponential distribution, Exponential integral; Gamma
distributions; Geometric distribution; Regeneration point;
Revealed faults; Unrevealed faults.

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Introduction

In recent papers, Phillips (1979, 1981a,b) has investigated a system consisting of two modules, faults in one of which can be revealed only on special inspection, while the other is monitored continuously. A fault developing in the first module is called 'unrevealed' (U) (until a special inspection is carried out), while a fault in the second module, which it is assumed will be detected immediately, is called 'revealed' (R). It is supposed that a special inspection for a U fault is carried out only when a R fault occurs. Time for inspection and repair will be neglected in our analysis, as in our initial paper on this subject (Kotz and Johnson (1984)) and in Phillips' work.

Phillips' model is based on three random variables: X, the time from repair of R to next occurrence of R assuming no U present; Y, the time from repair of U to next occurrence of U, assuming no R present; and Z, the time from a U fault to the next R fault. X and Y are assumed to be independent; Z is independent of X but may depend on Y.

Phillips supposes that both modules are repaired whenever an R fault occurs - whether there is a U fault or not. Consequently, after each repair, the system reverts to its initial state, so there is a regeneration point. Kotz and Johnson (1984) modified this model by supposing that only failed modules are repaired. When a U fault is found, after an R fault has called for a special inspection, both modules are repaired and the system returns to its initial state, (as it always does in the Phillips' model). If a U fault is not present this does not happen, so we cannot treat the occurrence of an R fault alone as a regeneration point.

In either model, dependence of Z on Y makes allowance for the fact that the second module is no longer new when the U fault occurs; the distribution of Z can also reflect the possible effects of increased stress on the second module when the first is not functioning properly.

In the present paper we will (i) derive the distribution of time (T, say) from initial state to discovery of a U fault; (ii) determine the expected proportion of time there is an unrepaired U fault in the first module; and (iii) obtain the distribution of the number of R faults up to and including the one leading to discovery of a U fault. As in our earlier work, time spent on repair will be ignored. (see Phillips (1981b, p. 658) for the justification of this assumption). A similar two-component model has been analysed by Chou and Butler (1983), assuming inspection to be perfect, in regard to detection of defects, but possibly costly in its effect on the modules. More recently, Murthy and Nguyen (1985) also introduce cost considerations in a model wherein there are no unrevealed faults, but occurrence of a fault in either of the modules may immediately cause a fault in the other (with known probabilities). Repair is immediate, and only faulty modules are repaired.

DISTRIBUTION OF TIME TO DISCOVERY OF A U FAULT

We will use the same notation as in Kotz and Johnson (1984) for distribution functions, using the notation $f_W(w)$ to denote the probability density function (PDF) of a random variable W, and

$$S_W(w) = \int_w^\infty f_W(t)dt$$

to denote its survival function.

We also introduce $f_n(x)$ to denote the PDF of the n-fold convolution of $f_X(x)$. Then

$$f_T(t) = \int_0^t S_X(y)f_Y(y)f_{Z|Y}(t|y)dy$$

$$+ \int_0^t \int_0^y \{ \sum_{n=1}^\infty f_n(\tau) \} S_X(y-\tau)f_Y(y)f_{Z|Y}(t-\tau|y-\tau)d\tau \, dy$$

(τ represents the time of occurrence of the last R fault <u>preceding</u> the R fault leading to discovery of the U fault, and $\int_a^b \{ \sum_{n=1}^\infty f_n(\tau) \}d\tau$ is the expected number of such events between times a and b.)

It would seem reasonable to take

$$f_{Z|Y}(z|y) = f^*(y+z)/S^*(y)$$

where $f^*(\cdot)$ is a PDF and $S^*(y) = \int_y^\infty f^*(x)dx$ is the corresponding survival distribution function. This representation allows for the age of the second module at the time of occurrence of the U fault in the first module, in a natural way (see e.g. Kotz and Johnson (1981)). As an illustrative example, take

$$f_X(x) = \theta^{-2} x e^{-x/\theta} \quad (x > 0; \ \theta > 0) \tag{1}$$

$$f^*(x) = \phi^{-2} x e^{-x/\phi} \quad (x > 0; \ \phi > 0) \tag{2}$$

(The expected values of these distributions are 2θ, 2ϕ respectively; usually we would have $\phi < \theta$.) For the present we do not specify $f_Y(y)$.

From (1), $f_n(x) = \theta^{-2n} x^{2n-1} e^{-x/\theta} / (2n-1)!$

$$S_X(x) = (1+\theta^{-1}x)e^{-x/\theta}; \quad S^*(x) = (1+\phi^{-1}x)e^{-x/\phi}$$

Also, $\sum_{n=1}^{\infty} f_n(x) = \theta^{-1} e^{-x/\theta} \sum_{n=1}^{\infty} \{(x\theta^{-1})^{2n-1} / (2n-1)!\}$

$$= \theta^{-1} e^{-x/\theta} \cdot \frac{1}{2}(e^{x/\theta} - e^{-x/\theta}) = \frac{1}{2} \theta^{-1}(1-e^{-2x/\theta})$$

Hence,

$$f_T(t) = \int_0^t (1+\theta^{-1}y)e^{-y/\theta} f_Y(y) \phi^{-2} t e^{-t/\phi}\{(1+\phi^{-1}y)e^{-y/\phi}\}^{-1} dy$$

$$+ \int_0^t \int_0^y \frac{1}{2} \theta^{-1}\left(1-e^{-2\tau/\theta}\right)\{1+\theta^{-1}(y-\tau)\}e^{-(y-\tau)/\theta} f_Y(y)$$

$$\cdot \phi^{-2}(t-\tau)e^{-(t-\tau)/\phi}[\{1+\phi^{-1}(y-\tau)\}e^{-(y-\tau)/\phi}]^{-1} d\tau \ dy$$

$$= \phi^{-2} t e^{-t/\phi} \int_0^t \frac{1+\theta^{-1}y}{1+\phi^{-1}y} f_Y(y) \cdot \exp\{-(\theta^{-1}-\phi^{-1})y\} dy$$

$$+ \frac{1}{2} \theta^{-1}\phi^{-2}e^{-t/\phi} \int_0^t \int_0^y (1-e^{-2\tau/\theta})$$

$$\cdot \frac{1+\theta^{-1}(y-\tau)}{1+\phi^{-1}(y-\tau)} (t-\tau)f_Y(y)\exp\{-(\theta^{-1}-\phi^{-1})y + \theta^{-1}\tau\} d\tau \ dy \tag{3}$$

The expression for $f_T(t)$ looks formidable, but can be simplified for some particular choices of $f_Y(y)$.

PROPORTION OF TIME THERE IS AN UNDISCOVERED U FAULT

At the time of the first R fault after occurrence of a U fault, both modules are repaired, and the system starts again from its initial state. So the log-run proportion of time there is an undiscovered U fault is the same as the expected proportion of time this is so in a single cycle to discovery of the <u>first</u> U fault. This is $E[Z]/E[T]$, where T denotes time to first discovery of a U fault. (Note that it is not $E[Z/T]$, which would give undue weight to the proportions for small values of T.)

We can, in principle, evaluate $E[T]$ from the PDF, $f_T(t)$, as

$$E[T] = \int_0^\infty t\, f_T(t)dt .$$

Since $Z = T-Y$, $E[Z] = E[T] - E[Y]$ and, of course $E[Y] = \int_0^\infty y\, f_Y(y)dy$.

In general, however, evaluation of $E[T]$ is difficult because of the complexity of $f_T(t)$ (see (3)). Considerable simplification results if Y has an exponential distribution

$$f_Y(y) = \omega^{-1}e^{-y/\omega} \qquad (y > 0;\ \omega > 0) \tag{4}$$

whereby we can avoid using $f_T(t)$ explicitly. Because of the lack-of-memory property of the exponential distribution, the system reverts to its initial state after <u>any</u> repair, and in particular after the first repair of an R fault, whether or not there is a U fault to be repaired at the same time. So we need only evaluate $E[Z']/E[T']$ where T' is the time to first occurrence of an R fault, and Z' is the time (which may be zero) there is a U fault before <u>any</u> R' fault occurs. We have

$$E[Z'] = \int_0^\infty \int_0^t S_X(y)f_Y(y)\{f^*(t)/S^*(y)\}(t-y)dy\, dt \tag{5}$$

and $E[T'] = \int_0^\infty \int_0^t S_X(y) f_Y(y) \{f^*(t)/S^*(y)\}\, t\, dy\, dt + \int_0^\infty t\, S_Y(t) f_X(t) dt .$

The first and second terms on the right-hand side correspond to the R fault being preceded, or not (respectively) by a U fault. Also from (5)

$$E[T'] = E[T] + \int_0^\infty y f_Y(y) \{S_X(y)/S^*(y)\} \int_y^\infty f^*(t) dt + \int_0^\infty t\, S_Y(t) f_X(t) dt$$

$$= E[T] + \int_0^\infty y \{f_Y(y) S_X(y) + S_Y(y) f_X(y)\}\, dy \qquad (6)$$

Inserting the PDF's from (1), (2) and (4), and putting $\theta^{-1} + \omega^{-1} = \beta^{-1}$ we have from (5)

$$E[T] = \int_0^\infty \int_0^t (1+\theta^{-1}y) e^{-y/\theta} \cdot \omega^{-1} e^{-y/\omega} \cdot \phi^{-2} t\, e^{-t/\phi} \{(1+\phi^{-1}y) e^{-y/\phi}\}^{-1} (t-y) dy\, dt$$

$$= (\omega\phi^2)^{-1} \int_0^\infty \frac{1+\theta^{-1}y}{1+\phi^{-1}y} \exp\{-(\beta^{-1}-\phi^{-1})y\} \int_y^\infty t(t-y) e^{-t/\phi} dt\, dy$$

$$= (\omega\phi^2)^{-1} \int_0^\infty \frac{1+\theta^{-1}y}{1+\phi^{-1}y} \cdot [2\phi^3\{1+\phi^{-1}y + \tfrac{1}{2}(\phi^{-1}y)^2\} - \phi^2 y(1+\phi^{-1}y)] e^{-y/\beta} dy$$

$$= \omega^{-1}\phi \int_0^\infty \frac{1+\theta^{-1}y}{1+\phi^{-1}y} (2+\phi^{-1}y) e^{-y/\beta} dy$$

$$= \omega^{-1}\phi \int_0^\infty (1+\theta^{-1}y + \frac{1+\theta^{-1}y}{1+\phi^{-1}y}) e^{-y/\beta} dy$$

$$= \omega^{-1}\phi \quad \{\beta + \theta^{-1}\beta^2 + \phi\theta^{-1}\beta + \phi(1-\phi\theta^{-1}) E_1(\phi\beta^{-1}) e^{\phi/\beta}\} \qquad (7)$$

where $E_1(w) = \int_1^\infty x^{-1} e^{-wx} dx$ is the exponential integral (see Appendix I).

Also, from (6), $E[T'] = E[T] + \int_0^\infty y\cdot\omega^{-1} e^{-y/\omega}(1+\theta^{-1}y) e^{-y/\theta} dy + \int_0^\infty t\, e^{-t/\omega}\theta^{-2} t\, e^{-t/\theta} dt$

$$= E[T] + \int_0^\infty \{\omega^{-1}y(1+\theta^{-1}y) + \theta^{-2}y^2\} e^{-y/\beta} dy$$

$$= E[T] + \omega^{-1}(\beta^2+2\theta^{-1}\beta^3) + 2\theta^{-2}\beta^3 = E[T] + \beta^2\theta^{-1}+\cdots .$$

Thus the ratio $\dfrac{E(Z')}{E(T')}$ is the same as $\dfrac{E(Z)}{E(T)}$.

The ratio $E[Z]/E[T]$ depends only on the ratios $\theta:\phi:\omega$. The following
table gives a few illustrative values. It should be remembered that
the expected values of the three distributions (1), (2) and (4) are
2θ, 2ϕ and ω respectively (not θ,ϕ and ω).

Table 1. $E[Z]/E[T]$ = Proportion of Time There is an Undiscovered U Fault

ω/θ	ϕ/θ			
	0.4	0.6	0.8	1.0
0.5	0.546	0.654	0.723	0.778
1	0.330	0.440	0.522	0.625
2	0.187	0.270	0.340	0.444
4	0.093	0.141	0.188	0.280
8	0.042	0.066	0.088	0.160

As is to be expected, the longer the expected life time of the first
module (ω) the less proportion of time there is an undiscovered U fault.
The greater the expected value of Z, given Y (i.e. the greater ϕ) the
greater the proportion of time there is an undiscovered U fault. The
rate of increase is more pronounced for larger values of ω/θ.

The model considered by Philips corresponds to our model with the
addition of the assumption that $f_{Z|Y}(z|y) = f_Z(z)$, independent of y.

DISTRIBUTION OF NUMBER OF R FAULTS UP TO AND INCLUDING DISCOVERY OF A U FAULT

Denoting the number of R faults up to and including discovery of a U
fault by N, we have

$$\Pr[N=n] = \int_0^\infty f_Y(y)\{S_n(y) - S_{n-1}(y)\}dy = E_Y[S_n(y) - S_{n-1}(y)] \qquad (8)$$

where $\quad S_n(y) = \int_y^\infty f_n(y)dy = 1 - \int_0^y f_n(y)dy$

Integrating by parts, we obtain the equivalent formula

$$\Pr[N=n] = \int_0^\infty S_Y(y)\{f_{n-1}(y) - f_n(y)\}dy \qquad (8)'$$

In particular, in the exponential case, with

$$f_Y(y) = \omega^{-1}e^{-y/\omega} \quad (y > 0; \; \omega > 0)$$

then because of the lack-of-memory property of the exponential distribution, the distribution of N is geometric with

$$\Pr[N=n] = P(1-P)^{n-1} \qquad (n=1,2,\ldots) \qquad (9)$$

where $P = \Pr[Y < X]$. Subject to the condition that the distribution of $Y(>0)$ is absolutely continuous, the converse is true. That is, if a distribution of form (9) for N holds, whatever be the density function $f_X(x)$, then Y must have an exponential distribution (cf. Grosswald et al (1980)). A proof is presented in Appendix II.

If X has the distribution (1) then

$$1 - P = \theta^{-2} \int_0^\infty x \; e^{-x/\theta}e^{-x/\omega} \; dx = \theta^{-2} \; \beta^2, \text{ so}$$

$$\Pr[N=n] = (1-\theta^{-2} \; \beta^2)(\theta^{-2} \; \beta^2)^{n-1} \qquad (n=1,2,\ldots)$$

In terms of ω, $\theta^{-2} \; \beta^2 = \theta^{-2}(\theta^{-1}+\omega^{-1})^{-2} = \omega^2(\theta+\omega)^{-2}$.

Formula (8) is quite general. For example, if

$$f_Y(y) = \omega^{-2} y \; e^{-y/\omega} \qquad (y > 0; \; \omega > 0) \qquad (10)$$

and X has distribution (1) then

$$Pr[N=n] = \omega^{-2} \int_0^\infty y \; e^{-y/\omega} \{\frac{(y/\theta)^{2n-1}}{(2n-1)!} + \frac{(y/\theta)^{2n-2}}{(2n-2)!}\} \; e^{-y/\theta} dy$$

$$= \omega^{-2} \{2n \theta^{-(2n-1)} \beta^{2n+1} + (2n-1)\theta^{-(2n-2)} \beta^{2n}\} \qquad (11)$$

$$= (\omega^{-1}\beta)^2 (\theta^{-1}\beta)^{2(n-1)} (2n\theta^{-1}\beta + 2n-1)$$

$$= (1-\theta^{-1}\beta)^2 (\theta^{-1}\beta)^{2(n-1)} (2n\theta^{-1}\beta + 2n-1) \qquad (n=1,2,\dots)$$

Using, in (1) and (10), general Erlang distributions, we can generate systematically, via (8), a family of discrete analogues of these distributions, of which (11) is a particular case.

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APPENDIX I

$$\int_0^\infty (1+at)(1+bt)^{-1}e^{-ct}dt = b^{-1}\int_0^\infty (1+ab^{-1}u)(1+u)^{-1}e^{-cu/b}du$$

$$= b^{-1}e^{\bar{c}/b}\int_1^\infty \{1+ab^{-1}(v-1)\}v^{-1}e^{-cv/b}dv$$

(with $v=1+u$)

$$= b^{-1}e^{c/b}\int_1^\infty \{ab^{-1}+(1-ab^{-1})v^{-1}\}e^{-cv/b}dv$$

$$= b^{-1}\{ac^{-1}+(1-ab^{-1})E_1(ab^{-1})e^{c/b}\} \ .$$

where $E_1(\omega) = -Ei(-\omega) = \int_1^\infty x^{-1}e^{-\omega x}dx$ is the exponential integral (tabulated, for example, in U.S. National Bureau of Standards (1941)).

Putting $a=\theta^{-1}$, $b=\phi^{-1}$ and $c=\beta^{-1}$ we obtain

$$\int_0^\infty \frac{1+\theta^{-1}t}{1+\phi^{-1}t}e^{-t/\beta}dt = \phi\{\beta\theta^{-1}+(1-\phi\theta^{-1})E_1(\phi\beta^{-1})e^{\phi/\beta}\}$$

This formula is used in equation (7).

APPENDIX II

We will show that if the distribution of N is of form (9), whatever the density function $f_X(x)$, and Y is absolutely continuous, then Y must have an exponential distribution.

Take the special case $f_X(x) = e^{-x}$ $(x > 0)$ whence $f_n(x) = x^{n-1}e^{-x}/(n-1)!$

$$\text{and } S_n(y) - S_{n-1}(y) = y^{n-1}e^{-y}/(n-1)!. \qquad (12)$$

If $g_1(y)$ and $g_2(y)$ are two different density functions of Y giving the same formula for $\Pr[N=n]$ then from (8) and (12)

$$\int_0^\infty \{e^{-y}g_1(y)\}y^{n-1}dy = \int_0^\infty \{e^{-y}g_2(y)\}y^{n-1}dy \quad (n=1,2,\dots)$$

This means that the two density functions $c_j e^{-y}g_j(y)$ $(0 < y)$ with $c_j = \{\int_0^\infty e^{-y}g_j(y)dy\}^{-1}$ $(j=1,2)$, have the same moments of positive integer order. Since the exponential density (e^{-y}) is determined by its moments, and $g_1(y), g_2(y)$ are density functions the density functions $c_j e^{-y}g_j(y)$ are *determined by their moments, whence*

$$c_1 e^{-y}g_1(y) \equiv c_2 e^{-y}g_2(y)$$

so $g_1(y) \equiv g_2(y)$ (and $c_1 = c_2$).

Since $g(y) = f_Y(y) = \omega^{-1}e^{-y/\omega}$ $(0 < \omega; 0 < y)$ does satisfy (9), it follows that $g_1(y)$ $(g_2(y))$ must be of this exponential form.

The result clearly holds under the broader conditions that (8) holds for some $S_n(y) - S_{n-1}(y) = y^{n-1}h(y)$ where $h(y)/\{\int_0^\infty h(y)dy\}$ is a density function determined by its moments.

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